

Implementation and Computational Evaluation of an Adaptive Cognitive Probability Distortion Model for Probabilistic Reasoning

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Abstract: The paper represents a computational implementation and analysis of the Adaptive Cognitive Probability Distortion Model (ACPDM), designed to present probabilistic reasoning changes under cognitive bias conditions. The implementation responses from formal probability, reasoning weight, heuristic distortion, decision response, Markov state transitions and Bayesian updating within a hybrid computational architecture. MATLAB has been used to analyse numerical patterns, sensitivity of parameters, learning priorities and cognitive-state interactions. The overall analysis defines how bias intensity and formal reasoning weight changes with obtained probability, whereas distributed learning approaches offer a mechanism by which heuristic distortion can persist or decrease. The analysis supports computational priority and offers a base for future simulation and empirical interpretation.

Keywords: ACPDM, probabilistic reasoning, mathematical modelling, MATLAB, cognitive bias, Bayesian updating, Markov transition, sensitivity analysis.

1. Introduction

Probabilistic reasoning can change according to the wider aspect of mathematical properties of a specific event and the cognitive processes have been referred to interpret. As proposed by Zhou et al. (2025), a computational channel model is significant when the objective is not only to calculate probability, but to present how probability may be changed throughout reasoning duration. ACPDM navigates this problem by interpreting normative probability and heuristic distortion activities along with a dynamic reasoning weight.

The implementation journey focuses on computational analysis and its execution. The model is changed within the mathematical activities, which can be executed under multiple parameter configurations. Apart from that, MATLAB offers the environment for numerical analysis, sensitivity benchmark, Markov transition analysis, dynamic learning practices, three-dimensional visualisation and Monte Carlo simulation (Latifi & Scoglio, 2025).

2. Computational Model Architecture

The computational structure supports sequential processes, while an objective event probability is justified as a normative reference. On the other hand, bias variables present gambler's fallacy, representativeness, base-rate neglect and anchoring (Gonzalez-Fernandez, 2024). Their weighted distribution supports a heuristic probability, whereas the formal reasoning weight estimates the normative contribution and reasoning to be obtained with probability.

The resulting probability is connected to a specific decision function, supporting probability to estimate a modelled response. The reasoning weight is analysed through cognitive error, learning gain and task difficulty, focusing on the model iteration. On the contrary, the Markov process indicates the movement between heuristic and formal states, while Bayesian updating changes beliefs after new evidence (Ortoleva, 2024). In this regard, this architecture includes probability distortion, decision-making, learning practices and cognitive transition within one concentrated sequence.

3. ACPDM Mathematical Implementation

3.1 Cognitive-State and Heuristic Representation

The implementation presents the cognitive conditions at the trial (t) as formal or heuristic reasoning. As argued by Oprea (2024), this distinction is particularly significant because heuristic estimates the changes from normative probability. In this regard, four different bias terms have been parameterised individually: representativeness, gambler's fallacy, anchoring and base-rate neglect. Teacher students often hold inaccurate or biased conceptions regarding randomness and probability. These conceptions reflect classroom and real-life related decision-making and reasoning criteria. This research study lacks formal frameworks that incorporate and stimulate these conceptions theoretically, especially with mathematical equations.

Therefore, this research study will navigate this gap by ensuring a structured model that will embed cognitive states of representation, decision functions and perceived probability. This model will involve explicit responses and modelling simulation to provide predictive insights into how reasoning patterns are integrated and how cognitive biases disrupt critical exploration.

3.2 Adaptive Reasoning Weight

The adaptive component facilitates a formal reasoning technique between one and zero. A greater value refers to a stronger dependency on formal probability, while a lower value indicates heuristic influence (Caetano et al., 2025). The weight is changed via learning gain, task difficulty level and cognitive error. Therefore, the model can represent deterioration or improvement within the formal reasoning technique rather than successive trials.

Weighting fractions will monitor the effect size. Lastly, simulation output will be statistically evaluated to analyse the result stability and outline recurring misconceptions. This methodology will not be dependent on the primary surveys. It will build a formal predictive model from secondary qualitative data like case study examples, secondary journals or online articles to offer both applied evaluation and a theoretical context of how student teachers' reasoning about probability and randomness.

3.3 Integrated Probability

The primary conceptual operation incorporates heuristic probability and objective based on the recent reasoning weight. As opined by Chua et al. (2024), holding objective probability shows consistency while changing a bias parameter or the formal reasoning weight supports the distortion to be analysed systematically. In addition to that, teacher perception programs are supposed to underspecify the probabilistic thinking requirement. As proposed by Shahzoda (2025), necessary programs explicitly navigate the specific reasoning, cognitive differences and metacognitive awareness, while the circular describes probability theory. As a result, it can influence how future teachers conceive and visualise probability and randomness. As illustrated by Williams-McBean (2024), considering the belief and understanding of how probability is described to the students, the teacher-conception gap can lead to misunderstanding in schooling.

Recent research studies have examined randomness involvement in both formal reasoning and probabilistic reasoning. Intuitive reasoning depends on pattern identification processes, similarity engagement and anecdotal evidence, while formal reasoning involves corporate sample space definition and frequency concepts. Various research studies have reported that preservice teachers exhibit mixed reasoning criteria. For instance, probabilistic preference and reasoning of prospective secondary school teachers in 2024 analysed approximately 66 learners, through computational probabilities (Álvarez-Arroyo, Batanero & Gea, 2024). They

struggled in elaborating media reports, correlation and confusing consideration regarding the conditional probability.

This conceptual framework illustrates how preservice teachers' reasoning about randomness and probability is analysed through a mathematical model, as shown in Figure 1. The model incorporates three dimensions: cognitive biases shaping perceptions, state transitions between heuristic and formal reasoning, and decision-making outcomes. Together, these components explain patterns of misconception and guide predictive insights.

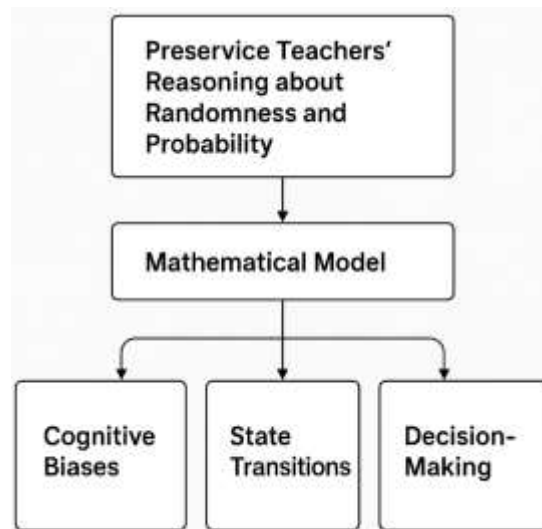


Figure 1: Conceptual Framework
(Source: Self-developed)

4. MATLAB Implementation

MATLAB changes the mathematical formulation within connected computational modules. The initial analysis considers ACPDM to obtain probability details.

```

Command Window
Objective Probability = 0.5000
Perceived Probability = 0.5615
Probability Distortion = 0.0615
>>
  
```

Figure 2: ACPDM command window
(Source: MATLAB)

Moving forward, the sensitivity module changes bias parameters and estimations of their effects.

The dynamic learning module changes the iteration based on the reasoning equation (Li et al., 2026).

Additionally, the Markov module estimates transition between heuristic and formal states.

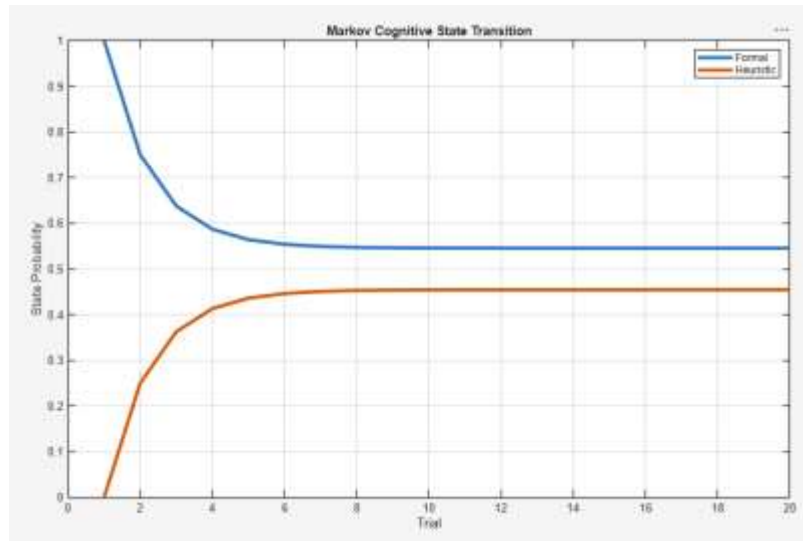


Figure 3: Markov cognitive state transition
(Source: MATLAB)

Lastly, the Monte Carlo simulation exceeds analysis throughout the repeated parameter configurations, whereas three-dimensional surface analysis supports visualisation interaction between variables.

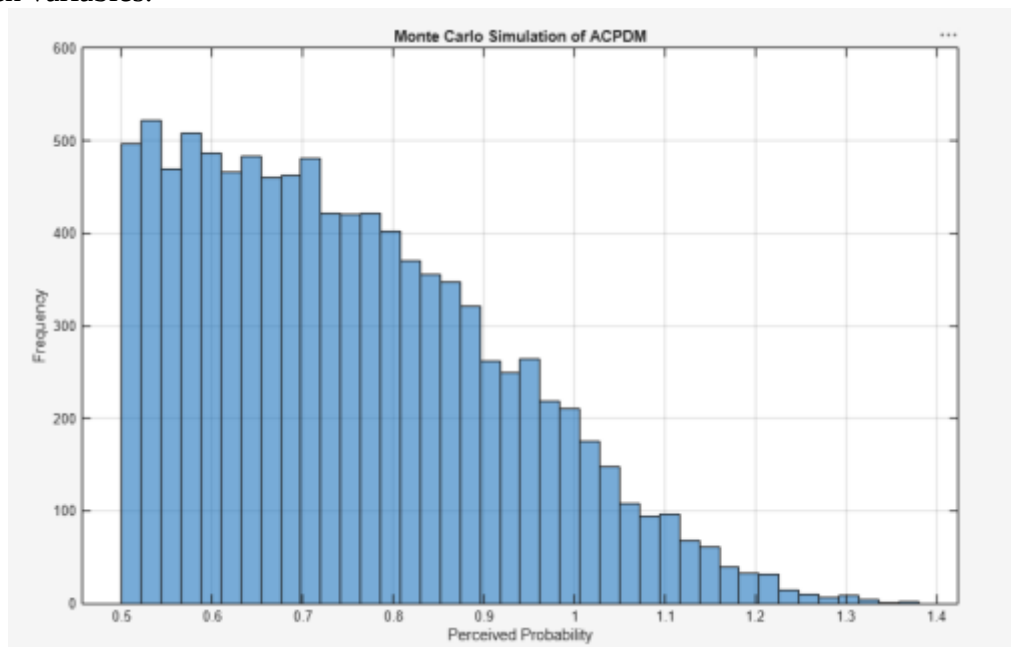


Figure 4: Monte Carlo simulation of ACPDM
(Source: MATLAB)

Selected MATLAB outputs have been represented as figures rather than producing individual screenshots. Sensitivity plotting, principal ACPDM output, learning trajectory, three-dimensional surface and Markov visualisation define the computational activities.

5. Computational Evaluation and Results

The numerical execution offers a direct identification of model measurements. Using the example, objective probability is 0.50, whereas the weighted bias configuration offers heuristic

estimation of 0.645. obtained probability becomes 0.558 with the formal reasoning rate of 0.60. Therefore, the difference from objective probability is 0.058 or 5.8% points. These figures reflect that defined cognitive parameters can transfer into measurable probabilistic changes.

On the other hand, sensitivity analysis offers another evaluation. For representativeness, sensitivity relies on the bias intensity and heuristic share. Within the formal reasoning weight (0.70) and representativeness intensity (0.40), the measured sensitivity is 0.12. It defines that parameter affects our conditional and the similar bias provides less distortion where formal reasoning shows rigidity.

Transition and dynamic simulations exceed the analysis across the multiple trials. Learning gain can enhance the formal reasoning weight, while task difficulty and cognitive error can neglect this movement. Therefore, the Markov component indicates movement towards formal probabilistic reasoning.

6. Critical Discussion

The computational analysis shows that ACPDM represents probabilistic modification as a parameter-sensitive and time-dependent procedure. Its primary advantages is the inclusion of a mechanism that would remain segmented such as bias intensity modifiers distortion, learning changes the weight, formal reasoning which manages its magnitude, Markov transformations indicate cognitive-state modification and Bayesian accommodate updated evidence. Additionally, MATLAB makes such relationships analytical through the controlled simulation processes (Bequette, 2026).

On the contrary, computational visibility is not an empirical analysis. The numerical representations have been specified with the model input apart from considering student-teacher observations (Yu et al., 2023). Therefore, the outputs illustrate behaviour employed by the assumptions and cannot support the real-world bias magnitudes or transition rates. Classroom data, empirical calibration and parameter estimation have been required before predictive claims can be examined.

7. Conclusion

The paper describes the computational execution of ACPDM as a hybrid model of probability reasoning under cognitive bias conditions. MATLAB allows numerical analysis, dynamic learning, sensitivity testing, simulation visualisation and Markov transitions. Furthermore, the numerical example ensures control differences between obtained probability and objective, whereas sensitivity analysis recommends the modelling of formal reasoning. The implementation supports computational visibility apart from its empirical selection.

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